National Defence
Research and Development Branch

Défense nationale
Bureau de recherche et développement

DREA CR/98/411

FURTHER ENHANCEMENT OF FREQUENCY RESPONSE AND DAMPING ANALYSIS CAPABILITIES OF THE **VAST** FINITE ELEMENT CODE (Phase III)
PART II: Development of an Out-of-Core Complex Eigenproblem Solver

L. Jiang — M.W. Chernuka

MARTEC Limited
Suite 400, 1888 Brunswick Street
Halifax, Nova Scotia, Canada
B3J 3J8

CONTRACTOR REPORT

Prepared for

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

Canada

THIS IS AN UNEDITED REPORT ON SCIENTIFIC OR TECHNICAL WORK CONTRACTED BY THE DEFENCE RESEARCH ESTABLISHMENT ATLANTIC OF THE RESEARCH AND DEVELOPMENT BRANCH OF THE DEPARTMENT OF NATIONAL DEFENCE, CANADA.

THE CONTENTS OF THE REPORT ARE THE RESPONSIBILITY OF THE CONTRACTOR, AND DO NOT NECESSARILY REFLECT THE OFFICIAL POLICIES OF THE DEPARTMENT OF NATIONAL DEFENCE.

PLEASE DIRECT ENQUIRIES TO:

THE DIRECTOR GENERAL
DEFENCE RESEARCH ESTABLISHMENT ATLANTIC
P.O. BOX 1012
DARTMOUTH, NOVA SCOTIA, CANADA
B2Y 3Z7

106-03146/

National Defence
Research and Development Branch

Défense nationale
Bureau de recherche et développement

DREA CR/98/411

FURTHER ENHANCEMENT OF FREQUENCY RESPONSE AND DAMPING ANALYSIS CAPABILITIES OF THE **VAST** FINITE ELEMENT CODE (Phase III) PART II: Development of an Out-of-Core Complex Eigenproblem Solver

L. Jiang — M.W. Chernuka
MARTEC Limited
Suite 400, 1888 Brunswick Street
Halifax, Nova Scotia, Canada
B3J 3J8

Accepted by : _____
D.C. Stredulinsky - Contract Scientific Authority

W7707-5-3261/01-HAL
Contract Number

September 1997

CONTRACTOR REPORT

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

Canada

ABSTRACT

An effective numerical method for solving complex eigenvalue problems, the complex inverse power method, is discussed in this report. The method is derived by extending its real version to handle complex stiffness matrices resulting from the finite element discretization of the viscoelastic damping materials. Details on the implementation of this method into the VAST program are then presented. Finally, the newly developed complex eigenproblem solver in VAST is verified by numerical problems.

RÉSUMÉ

Ce rapport examine une méthode numérique effective de résolution de problèmes complexes de valeur propre, la méthode de puissance inverse complexe. Cette méthode est obtenue en étendant sa valeur réelle pour traiter des matrices de rigidité complexes résultant d'une discrétisation par éléments finis des matériaux à amortissement viscoélastique. Des détails sur la mise en oeuvre de ce modèle dans le programme VAST sont ensuite présentés. Finalement, le résolveur de problèmes complexes de valeur propre utilisé dans VAST est vérifié par des problèmes numériques.

TABLE OF CONTENTS

ABSTRACT ii
TABLE OF CONTENTS iii
LIST OF TABLES iv
LIST OF FIGURES iv

1. INTRODUCTION 1.1

2. MATHEMATICAL DERIVATIONS OF THE COMPLEX INVERSE POWER METHOD 2.1

3. IMPLEMENTATION OF THE COMPLEX INVERSE POWER METHOD INTO THE VAST PROGRAM 3.1

4. NUMERICAL VERIFICATIONS 4.1

5. CONCLUSIONS 5.1

REFERENCES

APPENDIX A: MODIFIED VAST MANUAL FOR THE COMPLEX EIGENPROBLEM SOLVER

LIST OF TABLES

TABLE 4.1. Eigenvalues of the Cantilevered Plate Obtained by Using the Real and Complex Eigenproblem Solvers 4.3

LIST OF FIGURES

FIGURE 4.1. Problem Geometry and Finite Element Mesh for In-Plane Free Vibration Analysis of a Cantilevered Plate. 4.4

FIGURE 4.2. Problem Geometry and Finite Element Mesh for Free Vibration Analysis of a Cantilevered Infinite Sandwich Plate with a Constrained Damping Layer. 4.5

FIGURE 4.3. Variation of the Natural Frequencies of the Infinite Sandwich Plate with the Material Loss Factor (η_D) (a) Mode 1 ..... 4.6

FIGURE 4.3. (Continued). (b) Mode 2. ..... 4.7

FIGURE 4.3. (Continued). (c) Mode 3. ..... 4.8

FIGURE 4.3. (Continued). (d) Mode 4. ..... 4.9

FIGURE 4.4. Variation of the System Loss Factors (η_s) of the Infinite Sandwich Plate with the Material Loss Factor (η_D) (a) Mode 1. ..... 4.10

FIGURE 4.4. (Continued). (b) Mode 2. ..... 4.11

FIGURE 4.4. (Continued). (c) Mode 3. ..... 4.12

FIGURE 4.4. (Continued). (d) Mode 4. ..... 4.13

1. INTRODUCTION

With the finite element discretization, the governing equations for free vibration analyses of structures involving viscoelastic damping materials can be expressed as:

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 \tag{1.1}$$

where [K] is the complex stiffness matrix; [M] is the real mass matrix and {X} is the complex displacement vector. By assuming a solution of the form $\{X\} = \{\phi\}e^{ipt}$ for Equation (1.1), a complex eigenproblem is formulated as:

$$\left(p^2[M] - [K]\right)\{\phi\} = 0 \tag{1.2}$$

where p^2 and $\{\phi\}$ are the complex eigenvalue and complex eigenvector, respectively. Following Rao's notation [1], the system loss factor η is related to p^2 via the following expression:

$$p^2 = p_r^2 + ip_r^2\eta \tag{1.3}$$

where p_r and η are real numbers. Setting $\lambda = p^2$, the eigenvalue problem defined in Equation (1.2) can be put into a more general form as:

$$[K]\{\phi\} = \lambda[M]\{\phi\} \tag{1.4}$$

Since the stiffness matrix, the eigenvalues and the eigenvectors are all complex, the solvers for the eigenvalue analysis in the finite element program VAST [2] of the current version cannot be directly applied to solve Equation (1.4). Some new solvers, which have the capability of solving the complex eigenproblem are required. As a result, a complex inverse power method, which is analogous to the real inverse power method [3], is developed in this report. This choice has been based on the fact that this method is among the most effective algorithms for eigenvalue analysis, provided that a small number or portion of eigenvalues and eigenvectors are needed to be extracted from the finite element discretized system, and its real version is already available in VAST.

2. MATHEMATICAL DERIVATIONS OF THE COMPLEX INVERSE POWER METHOD

Assume that a shift μ is applied to accelerate the iteration process, the eigenproblem solved becomes:

$$([K] - \mu[M])\ \{\phi\} = \nu[M]\{\phi\} \tag{2.1}$$

where the eigenvalues of the original problem (1.4) and the shifted eigenproblem in Equation (2.1) are related by $\nu_i = \lambda_i - \mu$, i=1,...,n; n is the total number of degrees-of-freedom of the structure. Let $[\hat{K}] = [K] - \mu[M]$, Equation (2.1) can also be written as:

$$[\hat{K}]\{\phi\} = \nu[M]\{\phi\}\ . \tag{2.2}$$

Comparing Equation (2.2) and Equation (1.4), it can be seen that these two equations are very similar. The eigenvalue analysis algorithms applied to Equation (1.4) can also be applied to Equation (2.2). For the sake of simplicity for the presentation, only the solution procedure for Equation (1.4) is presented here.

The complex inverse power method is an iterative method. It starts with a trial vector $\{X\}_0$, where every component is set equal to unity, i.e. (1+0i). Then $\{X\}_0$ is normalized in accordance with the following expression:

$$\{X\}_0 \Leftarrow \frac{\{X\}_0}{\left(\{X\}_0^T[M]\{X\}_0\right)^{1/2}}\ . \tag{2.3}$$

The next iteration vector, i.e., $\{X\}_1$ is obtained by solving the following linear equation system:

$$[K]\{X\}_1 = [M]\{X\}_0 \ . \tag{2.4}$$

$\{X\}_1$ must also be normalized by an equation similar to Equation (2.3) before the next iteration. For k=0,1,2,..., we have:

$$\{X\}_k \Leftarrow \frac{\{X\}_k}{\left(\{X\}_k^T[M]\{X\}_k\right)^{1/2}} \tag{2.5}$$

and

$$[K]\{X\}_{k+1} = [M]\{X\}_k \tag{2.6}$$

where it is assumed that $\{X\}_{k+1}^T[M]\{X\}_{k+1} \neq 0$. When k becomes a large number, the first eigenvalue λ_1 and the eigenvector $\{\phi_1\}$ are obtained as:

$$\frac{1}{\lambda_1} = \left(\{X\}_{k+1}^T[M]\{X\}_{k+1}\right)^{1/2} , \tag{2.7}$$

and

$$\{\phi_1\} = \lambda_1\{X\}_{k+1} \ . \tag{2.8}$$

The convergence criteria for the above iteration process is:

$$\frac{|\lambda_1^{(k+1)} - \lambda_1^{(k)}|}{\lambda_1^{(k+1)}} \leq \text{tol} \tag{2.9}$$

where the superscripts represent the iteration numbers and tol is a prescribed small positive real number.

To obtain the eigenvalues and the eigenvectors for the higher modes of free vibration, i.e., λ_i and $\{\phi_i\}$, i=2,3,..., iterations are carried out using the following equations:

$$\{X\}_0 = \{\phi_{i-1}\} \tag{2.10}$$

$$[K]\{X\}_{k+1} = [M]\{X\}_k \tag{2.11}$$

$$\{Y\}_{k+1} = \{X\}_{k+1} - \sum_{j=1}^{i-1} \alpha_j \frac{1}{\lambda_j}\{\phi_j\} \tag{2.12}$$

$$\{X\}_{k+1} = \frac{\{Y\}_{k+1}}{\left(\{Y\}_{k+1}^T [M]\{Y\}_{k+1}\right)^{1/2}} \tag{2.13}$$

where $\alpha_j = \{\phi_j\}^T [M]\{X\}_k$ and k=0,1,2,... For each iteration, the ith eigenvalue and eigenvector can be computed approximately in the same manner as in Equations (2.7) and (2.8) and the iteration process is terminated when the convergence criterion for λ_i is satisfied as:

$$\frac{|\lambda_i^{(k+1)} - \lambda_i^{(k)}|}{\lambda_i^{(k+1)}} \le tol\ . \tag{2.14}$$

3. IMPLEMENTATION OF THE COMPLEX INVERSE POWER METHOD INTO THE VAST PROGRAM

In order to implement the complex inverse power method into the VAST program to solving complex eigenvalue problems, three new modules have been created. They are ASSEM8, DECOM8 and EIGEN7.

ASSEM8 performs the assembly of the global stiffness and mass matrices. As mentioned earlier, in free vibration analyses of viscoelastically damped structures, the global stiffness matrix is complex and the imaginary part of the stiffness depends on the material loss factors of the damping materials. In ASSEM8, the user is provided an opportunity to input the material loss factors for selected or all element groups. The real part of the stiffness matrix and the mass matrix are still stored in exactly the same format on file T46 as in a real eigenvalue analysis, whereas the imaginary part of the stiffness matrix is stored behind the mass matrix, similar to the geometric stiffness matrix. Since ASSEM8 was developed based on module ASSEM1, many of the useful features in ASSEM1, such as the bandwidth reduction and the penalty method for treating multipoint constraints (MPC), were still preserved. The biggest advantage associated with this T46 format is that modifications in modules STIFM and MASSM are completely avoided in the implementation of the complex eigenvalue analysis capability in VAST.

DECOM8 decomposes the complex effective stiffness matrix and was developed based on the existing module DECOM1. In DECOM8, the real and imaginary parts of the stiffness matrix are retrieved from file T48, and then combined and arranged in block form. A newly developed subroutine, DCOMPC, is subsequently called to perform the LU decomposition of the complex matrix. As for real stiffness matrices, the decomposed matrix is stored on T80, whereas some related control parameters are kept on T50.

EIGEN7 is a complex version of EIGEN1 which extracts eigenvalues and eigenvectors of a finite element discretized system. In addition to changing definitions of variables and built-in functions from REAL to COMPLEX, several subroutines have to be modified to allow complex

arithmetic operations. The section of code for linearized buckling analysis now becomes irrelevant and has been eliminated from this module. The default output to the LPT file includes the mode number, natural frequency, system loss factor and the number of iterations. Because the system loss factor can be obtained directly from the complex eigenvalues, the strain energy ratios (SERs) are no longer required.

The input data required by these modules are summarized in Appendix A.

4. NUMERICAL VERIFICATIONS

In order to validate the implementation of the complex inverse power method for the complex eigenvalue analysis, two example problems were analyzed, using the new VAST program.

The first example problem considered is a plate fixed at one end and subjected to in-plane free vibration, as shown in Figure 4.1(a). The length, width and thickness of the plate were 8 m, 2 m and 1 m, respectively, and it was treated as a plane stress problem in the present analysis. The Young's modulus, Poisson's ratio and mass density of the material were taken to be 2.06×10^{11} N/m^2, 0.3 and 7.87×10^3 kg/m^3. The finite element mesh used is shown in Figure 4.1(b), with a total of 16 eight-node membrane elements. The verification process contained three steps. In the first step, a real eigenvalue analysis was carried out to obtain the four smallest eigenvalues, which are listed in Table 4.1. In the second and the third steps, complex eigenvalue analyses were carried out with the stiffness matrices as:

$$[K]_{C1} = [K]_R + i[0] \tag{4.1}$$

and

$$[K]_{C2} = (1+i)[K]_R \tag{4.2}$$

respectively, where $[K]_R$ is the stiffness matrix used in the real eigenvalue analysis. The results of the complex eigenvalue analyses are also contained in Table 4.1. For [K] in Equation (4.1), it is expected that the real parts of the complex eigenvalues are equal to the eigenvalues obtained in the real eigenvalue analysis, and the imaginary parts of the complex eigenvalues are zero. For [K] in Equation (4.2), the real and imaginary parts of the eigenvalue are expected to be identical and equal to the eigenvalue obtained in the real eigenvalue analysis. This is because when $[K]_R\{\phi\} = \lambda[M]\{\phi\}$, the following equation is also satisfied:

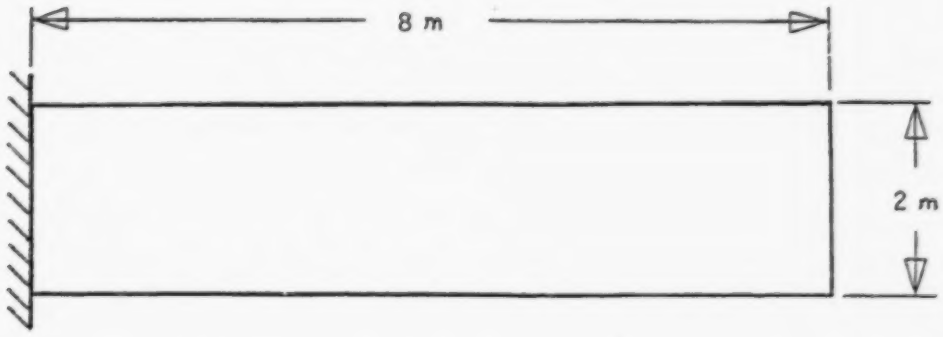
$$(1+i)[K]_R\{\phi\} = (1+i)\lambda[M]\{\phi\} \tag{4.3}$$

By comparing the results of the real and complex eigenvalues in Table 4.1, it can be seen that the obtained complex eigenvalues agree precisely with the expected results.

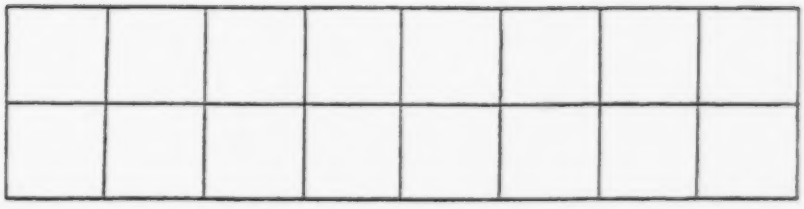
The second example problem is the free vibration of a cantilevered infinite sandwich plate. This plate was 7" long and composed of two identical 0.06" thick aluminium plates and a very thin layer (0.005") of viscoelastic damping material. The cross-section of the plate and its plane strain finite element model are shown in Figure 4.2. The Young's modulus, Poisson's ratio and mass density of the core material were assumed to be 2.817×10^4 psi, 0.49 and 0.7×10^{-4} lbs-sec^2/in^4, respectively. To characterize the dependence of the natural frequency and system loss factor (η_S) on the material loss factor of the core (η_D), a parametric study was carried out in which the first four complex eigenvalues were obtained for a set of material loss factors ranging from 0.0 to 1.0. The results of real and complex eigenvalue analyses are graphically displayed in Figures 4.3 and 4.4. As indicated in these figures, the natural frequencies of the complex system increase with the increase of the material loss factor, whereas the ratio of the system and material loss factors, η_S/η_D, decreases. These results not only demonstrated the highly nonlinear relationship between the damping material property and global structural response, but also verified the complex eigensolver because the real eigensolution was found to be a linearization of the complex one at a very small material loss factor. A number of direct frequency response analyses were also performed using constant stiffness and damping properties for the core material. The system loss factors predicted by the complex eigensolution and frequency response analysis are in excellent agreement.

TABLE 4.1. Eigenvalues of the Cantilevered Plate Obtained by Using the Real and Complex Eigenproblem Solvers

Mode No.	EIGEN1	EIGEN7	
	$[K]_R$	$[K]_{C1}$	$[K]_{C2}$
1	2.4284×10^4	2.4284×10^4 (1+0i)	2.4284×10^4 (1+i)
2	6.3025×10^5	6.3025×10^5 (1+0i)	6.3025×10^5 (1+i)
3	1.0151×10^6	1.0151×10^6 (1+0i)	1.0151×10^6 (1+i)
4	3.3461×10^6	3.3461×10^6 (1+0i)	3.3461×10^6 (1+i)

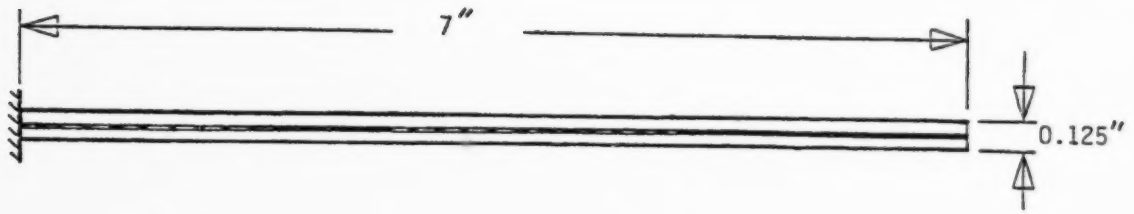


(a) Problem Geometry



(b) Finite Element Mesh

FIGURE 4.1. Problem Geometry and Finite Element Mesh for In-Plane Free Vibration Analysis of a Cantilevered Plate.



(a) Problem Geometry

(b) Finite Element Mesh

FIGURE 4.2. Problem Geometry and Finite Element Mesh for Free Vibration Analysis of a Cantilevered Infinite Sandwich Plate with a Constrained Damping Layer.

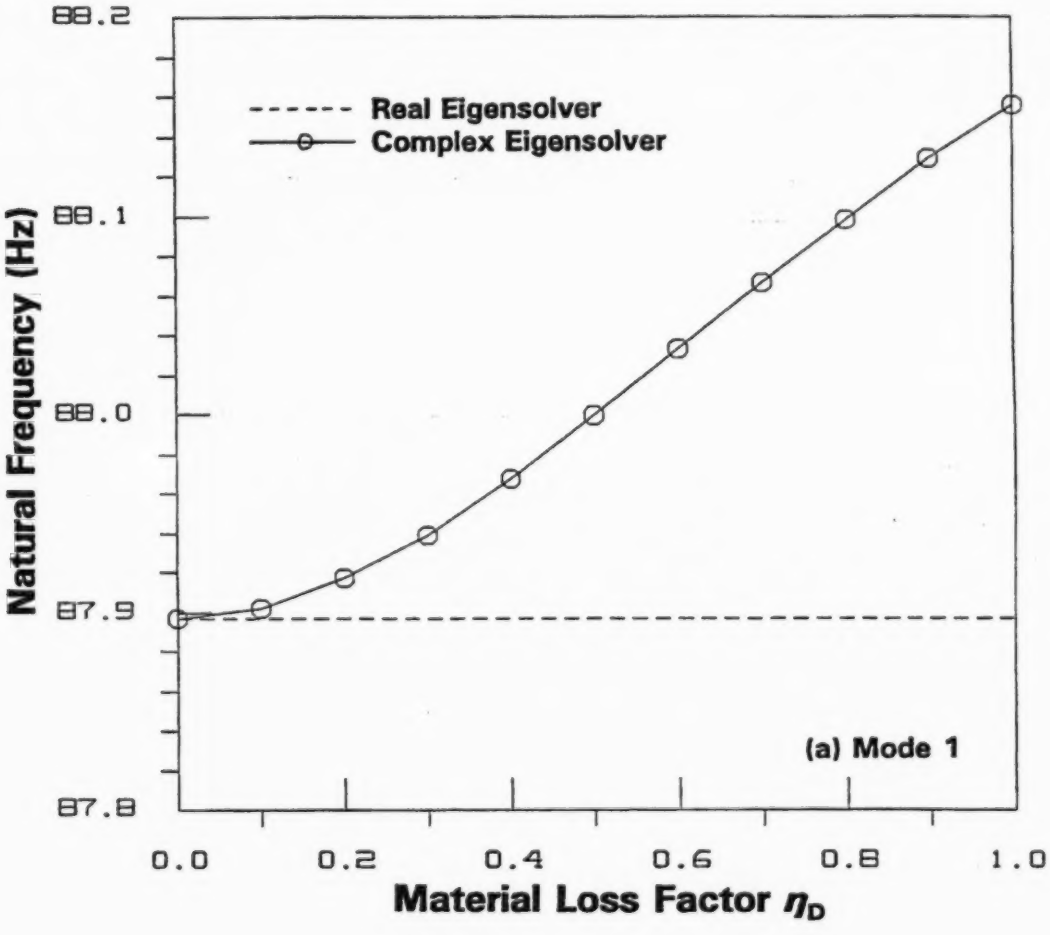


FIGURE 4.3. Variation of the Natural Frequencies of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (a) Mode 1.

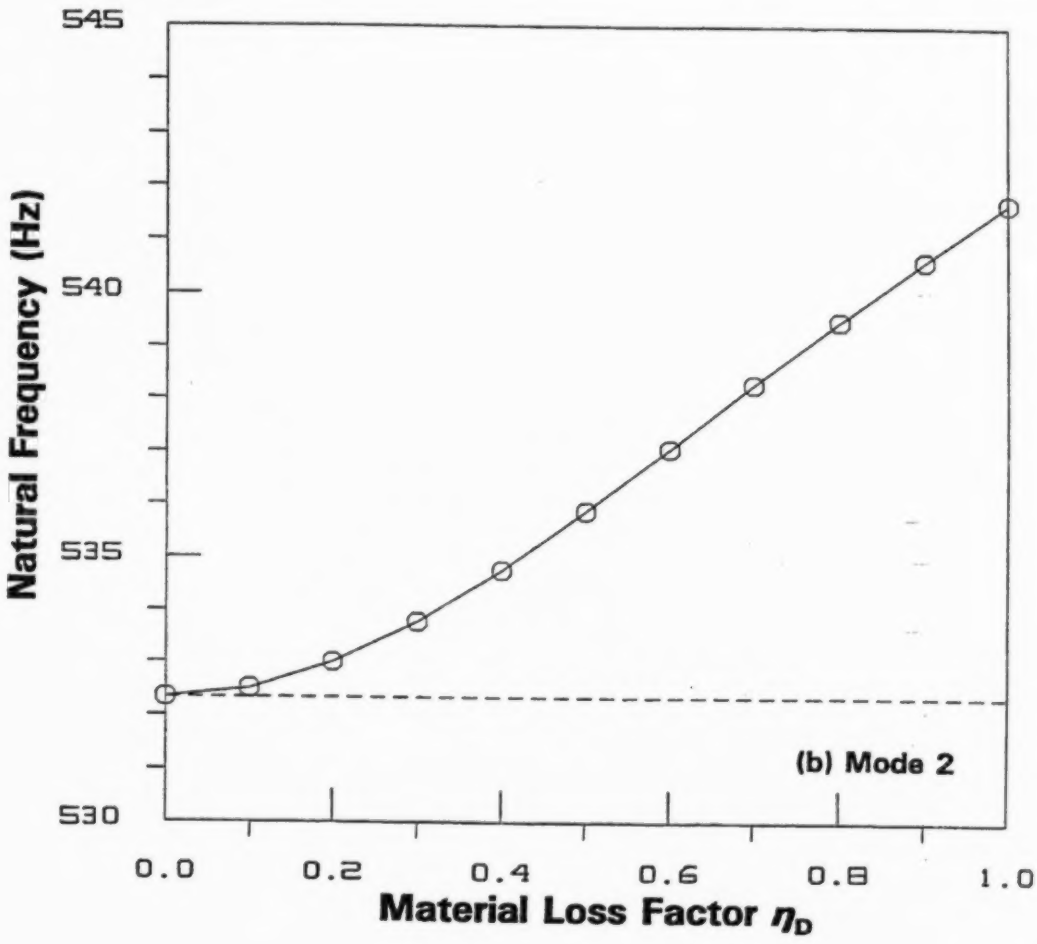


FIGURE 4.3 (Continued). Variation of the Natural Frequencies of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (b) Mode 2.

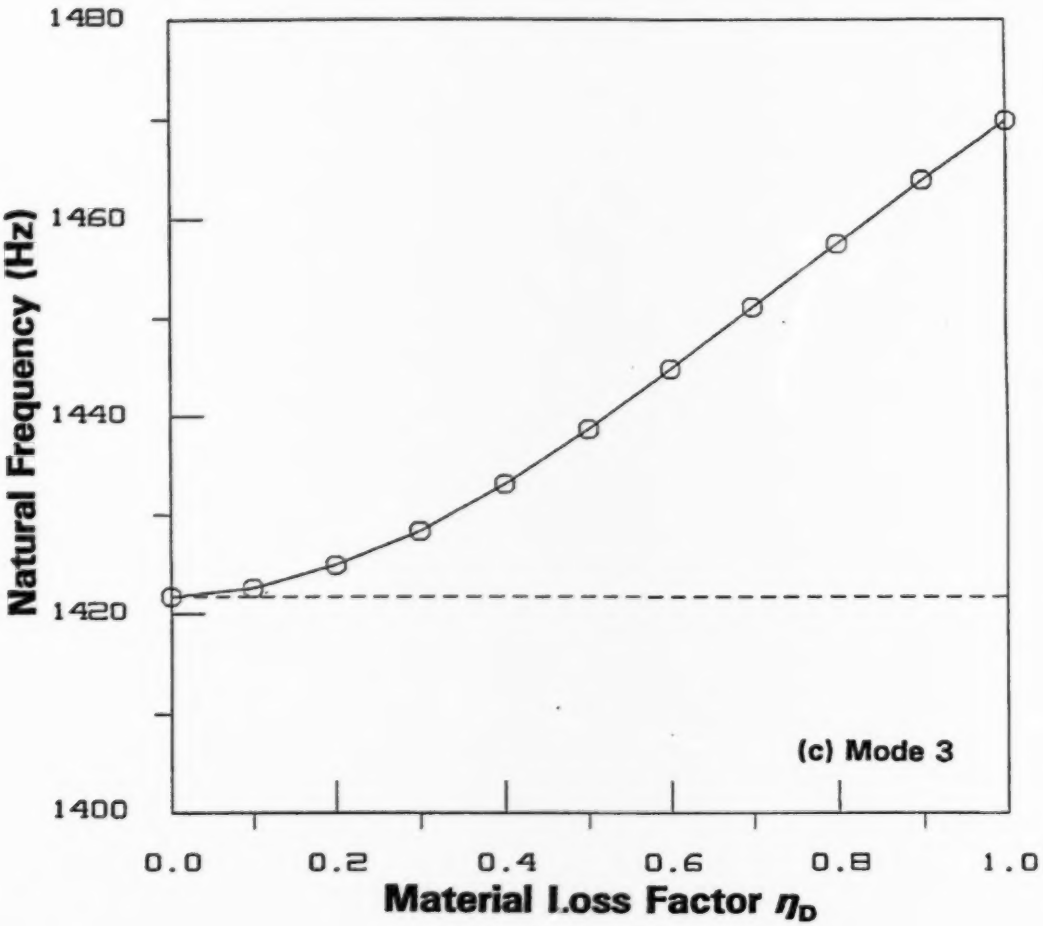


FIGURE 4.3 (Continued). Variation of the Natural Frequencies of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (c) Mode 3.

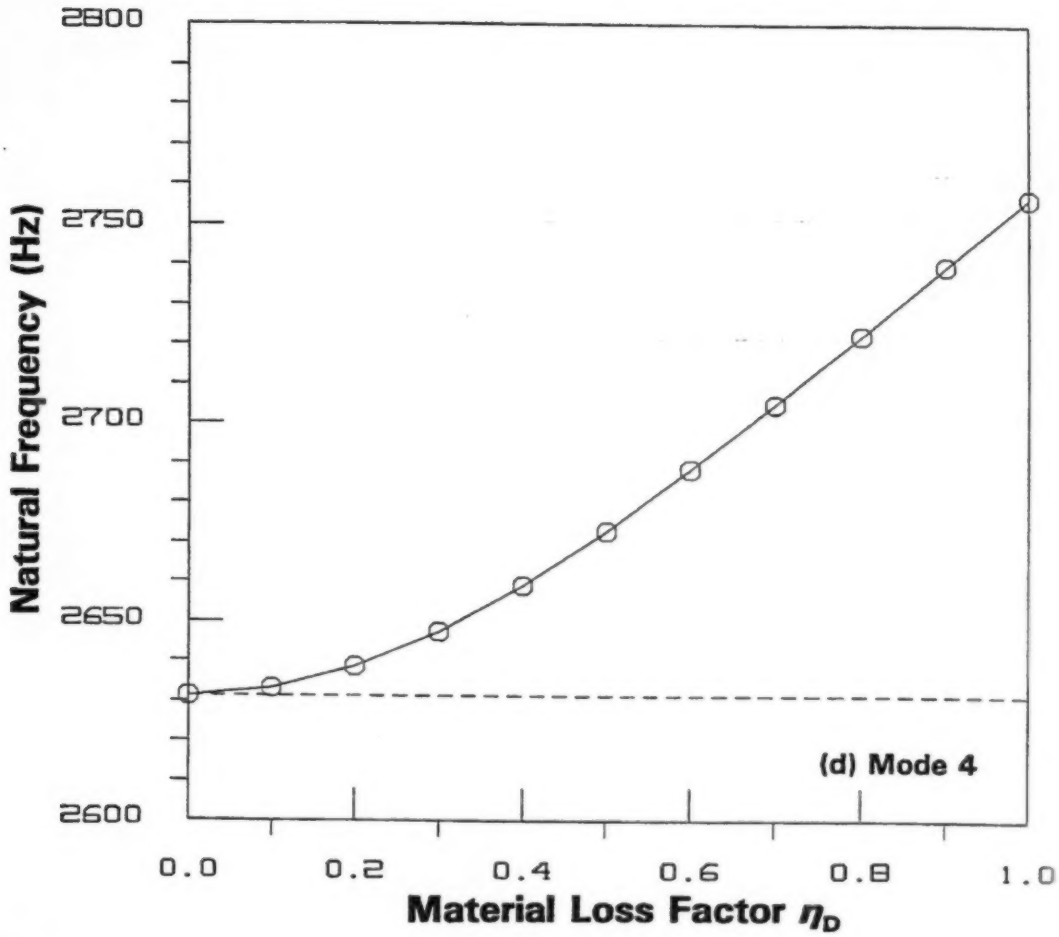


FIGURE 4.3 (Continued). Variation of the Natural Frequencies of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (d) Mode 4.

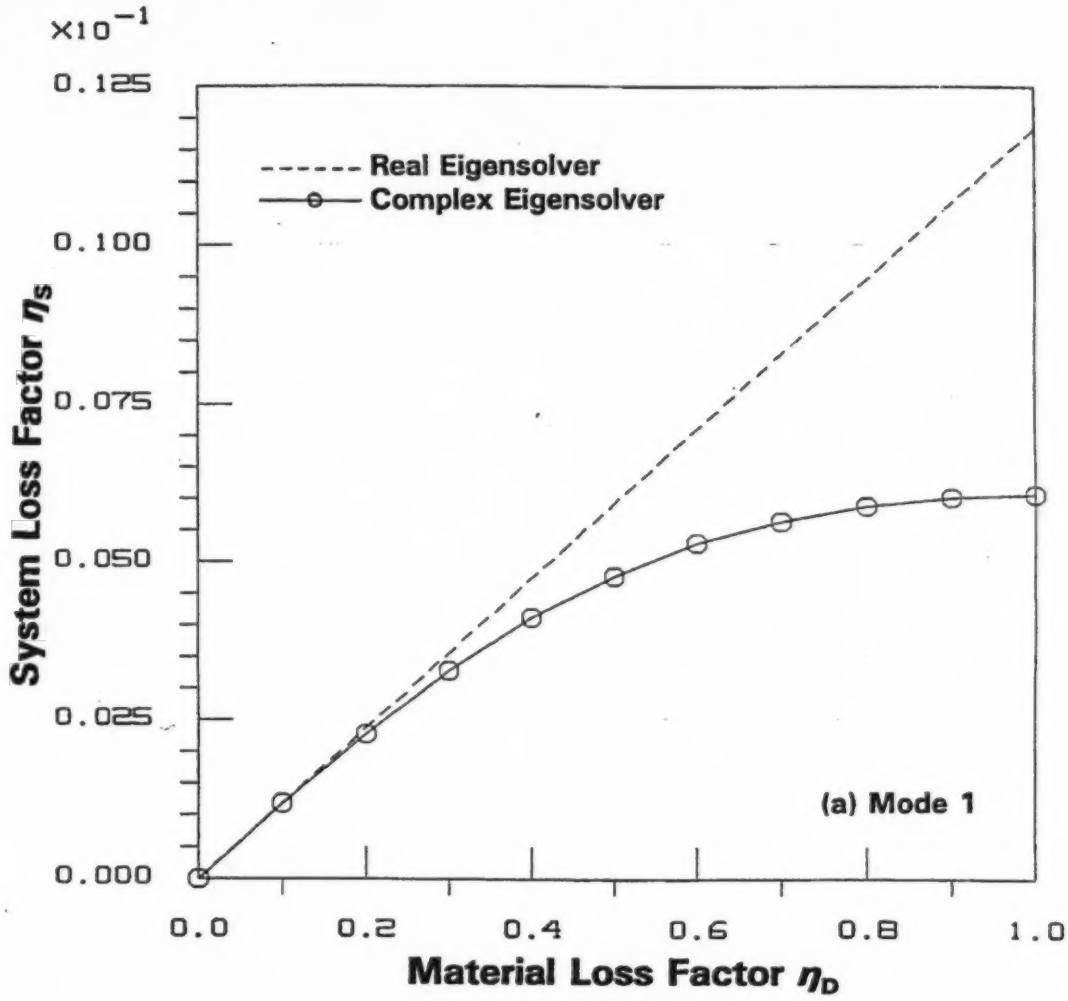


FIGURE 4.4. Variation of the System Loss Factors (η_s) of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (a) Mode 1.

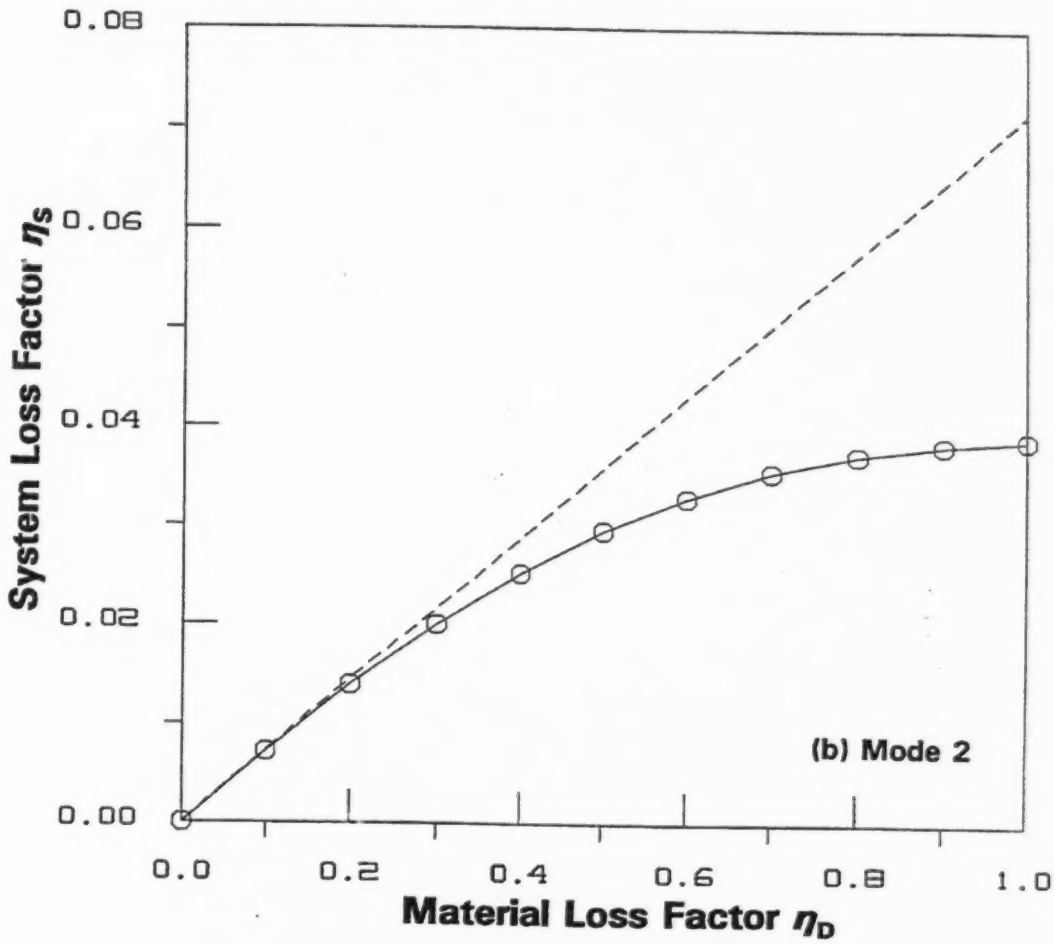


FIGURE 4.4 (Continued). Variation of the System Loss Factors (η_s) of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (b) Mode 2.

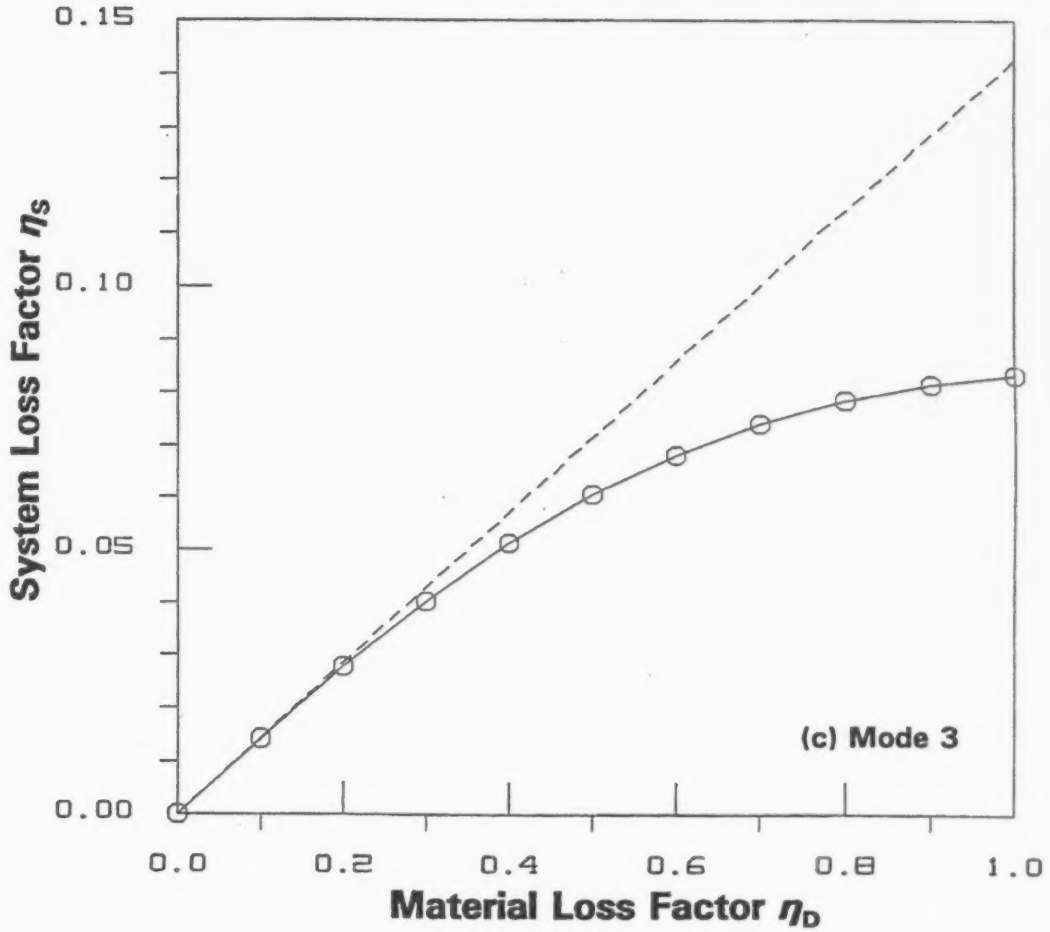


FIGURE 4.4 (Continued). Variation of the System Loss Factors (η_s) of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (c) Mode 3.

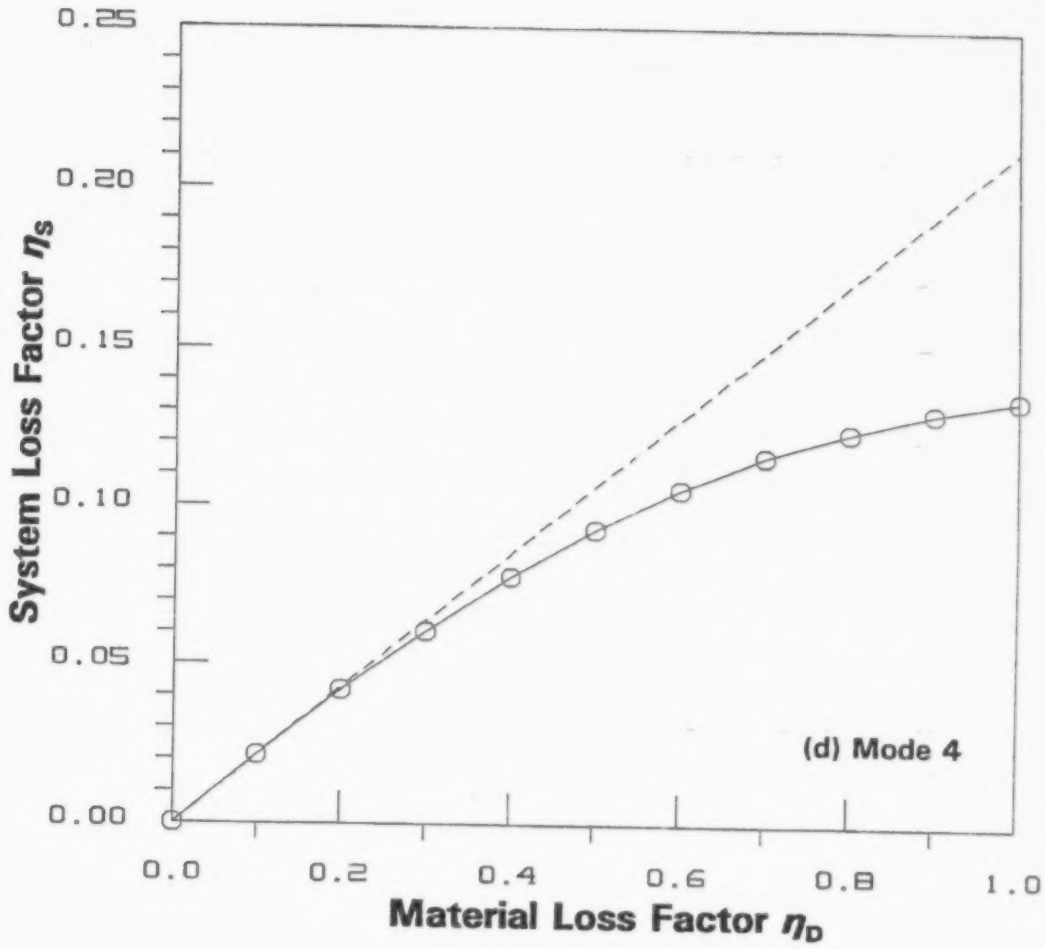


FIGURE 4.4 (Continued). Variation of the System Loss Factors (η_s) of the Infinite Sandwich Plate with the Material Loss Factor (η_D). (d) Mode 4.

5. CONCLUSIONS

A complex inverse power method for the computation of the complex eigenvalues and eigenvectors, resulting from the complex stiffness in the finite element analysis of viscoelastically damped structures has been presented. The implementation of this method into the finite element program VAST was carried out. This complex eigenvalue analysis capability was verified by the example problems.

It should be mentioned that the eigenvalues of equal modulus [5] were not dealt with in this report. If such eigenvalues occur in the structural analysis, the formulation of complex inverse power method presented should be modified by using the approach outlined in Reference [5]. By using the complex subspace method, the convergence problem arising from the eigenvalues having equal modulus can be avoided. Therefore the complex subspace iteration method is more efficient for general purpose structural analysis and is therefore recommended for implementation into the VAST program in the future.

REFERENCES

[1] D.K. Rao. "Frequency and Loss Factors of Sandwich Beams Under Various Boundary Conditions". J. Mechanical Engineering Science, pp. 271-282, Vol. 20, 1978.

[2] VAST User's Manual, Version #7.1, Martec Ltd., Halifax, Nova Scotia, 1995.

[3] K.J. Bathe. "Finite Element Procedures in Engineering Analysis". Prentice-hall, Englewood Cliffs, New Jersey, 1982.

[4] K.J. Bathe and E.L. Wilson. "Solution Methods for Eigenvalue Problems in Structural Mechanics". Int. J. for Numerical Methods in Engineering, pp. 213-226, Vol. 6, 1973.

[5] A. Jennings and J.J. McKeown. "Matrix Computation, Second Edition". John Wiley & Sons, Chichester, 1992.

APPENDIX A

MODIFIED VAST MANUAL FOR THE COMPLEX EIGENPROBLEM SOLVER

Card 1 (1115)

IELEMS = 0, restart (element matrices have been computed during a previous run).

= 1, element matrices are computed (both regular elements and superelements if substructuring is used).

= 2, only regular element matrices are computed. This applies only when substructuring is used, and permits the user to compute regular elements and superelements in two steps. The feature permits geometry and regular element data to be checked prior to the more costly computation of superelement matrices.

= 3, restart for computing superelement matrices. This applies only when substructuring is used, and assumes that a previous run with IELEMS = 2 has been made.

= 4, groups of substructure and superelement files have been generated by previous executions and are to be combined into one group of files.

IBANRD ≠ 0, nodes are renumbered to reduce bandwidth (for sub structure analysis, the highest level master nodes).

Note: This step can be performed during assembly if default values for Section C4 data are to be used.

IASSEM ≠ 0, element stiffness and mass matrices are assembled (for substructure analysis, this applies to superelement matrices).

= 1, for regular analysis.
= 2, for fixed-interface component mode synthesis method.
= 3, for free-interface component mode synthesis method.
= 8, for complex eigenvalue analysis.
= 0, for improved solution algorithm (IDECOM = 3).

ISTIFM ≠ 0, stiffness modification step is activated to include boundary conditions and/or stiffness additions and/or multi-point constraints.

Note: This step must be performed for all types of analysis. If there are no stiffness modifications to be made, it must be indicated in the input data of Section C6, not by setting ISTIFM = 0. In the free-interface component mode synthesis method, ISTIFM must be set to 0. If the improved solution algorithm is employed (IDECOM = 3), ISTIFM also must be set to 0.

IMASSM ≠ 0, mass modification step is activated to include lumped masses and/or fluid added mass.

Note: This step must be performed for all types of analysis requiring mass (see Note 2, Section C2). If there are no mass modifications to be made, it must be indicated in the input data of Section C7, not by setting IMASSM = 0. For the free-interface component mode synthesis method, IMASSM must be set to 0.

= 1, the fluid added mass (if to be included) is assumed to be available on disk file PREFX.T36.

= 2, the fluid added mass is generated by VAST using the fluid finite element method. Data input described in Section D must be provided by the user. The fluid added mass is stored on disk file PREFX.T36.

= 3, same as IMASSM = 2, except that execution of VAST is terminated as soon as disk file PREFX.T36 has been created.

= 4, 5, 6, or 7, same as IMASS = 3, except that the four fluid added mass modules are executed one at a time, i.e., IMASSM = 4, 5, 6, and 7 causes, respectively, the modules ADMAS1, ADMAS2, ADMAS3 and ADMAS4 to be executed.

= 8, the fluid added mass is generated by VAST using the surface panel method. Data input described in Section D must be provided by the user. The fluid added mass is stored on disk file PREFX.T36.

= 9, same as IMASSM = 8, except that the execution of VAST is terminated as soon as disk file PREFX.T36 has been created.

IDECOM = 1, matrix decomposition is performed using module DECOM1, which assumes that the global stiffness and/or mass matrices have been properly assembled and modified. A matrix decom-

position is required for all types of analyses. However, the form of the matrix to be decomposed depends on the type of analysis.

= 3, matrix assembly, modification and decomposition are performed using the improved solution algorithm. For this option, ISOLVE must be 1.

= 8, decomposition of complex matrix is performed using module DECOM8.

IEIGEN ≠ 0, eigenvalue analysis is performed (see Notes in Section C9).

= ±1, natural frequencies or buckling load determined via direct iteration method. If negative, wet (natural frequency) modes will be computed separately using dry modes (see Note 4, Section C7).

= ±2, natural frequencies or buckling load determined via subspace iteration method. If negative, wet (natural frequency) modes will be computed separately using dry modes (see Note 4, Section C7).

= 3, restart for computation of wet (natural frequency) modes separately, using dry modes from a previous eigenvalue analysis (see Note 4, Section C7).

= 4, restart for Sturm sequence for missing eigenvalues (natural frequency analysis only).

= 5, restart for superelement eigenvector recovery.

= 6, numerical error estimation (see Section B8.B).

= 7, complex eigenvalue analysis via direct iteration method.

ILOADS ≠ 0, load vectors for static, dynamic, or frequency response analysis are computed.

= ±1, element/concentrated. If negative, the follower force effect is accounted for in element loads (for geometrically nonlinear analysis).

= 2, support motion (translational accelerations).

= 4, restart for superelement load generation.

= 5, restart for load case combinations.

= 9, random vibration

IDISPS ≠ 0, nodal point displacements are computed (see Notes in Section C11).

= ±1, static displacements. If negative, restart for harmonic component displacement combination when axisymmetric elements are used (NHAR≠0).

= ±2, dynamic displacements via modal superposition. If negative, restart for harmonic component displacement combination when axisymmetric elements are used (NHAR≠0).

= ±3, dynamic displacements via direct integration. If negative, restart for harmonic component displacement combination when axisymmetric elements are used (NHAR≠0).

= 4, response spectrum.

= 5, modal frequency response.

= 6, direct frequency response.

= 7, restart for superelement displacement recovery.

ISTRES ≠ 0, element stresses are computed.

IPOSTP ≠ 0, post-processing features are used.

= 1, post-processing of nodal stresses and/or strains.

= 2, random vibration analysis using modal or direct method.

= 3, combination of 1 and 2.

= 4, quasi-static random vibration.

C5.E Assembly of Mass and Complex Stiffness Matrices

Header Card (A6)

HEADR = 6 character literal constant 'IASEM8'.

Card 1 (3I5)

IBRC = 0, bandwidth reduction performed using default values (see Section C4, Note 4).

= 1, bandwidth reduction has been performed via Section 4 (IBANRD = 1).

= 2, bandwidth reduction is not used.

ICOL = 1, matrices will be assembled columnwise (see Note 4). This option must be used in VAST analyses.

= 0, matrices will be assembled rowwise. This option must NOT be used in VAST analysis.

IPTC = 0, printout of input data and warning message is suppressed.

= 1, input data for multipoint constraints is printed.

= 2, warning messages for disparity check are also printed.

Card 2 (2I5)

NDAMPM = number of element groups with nonzero material loss factors.

Provide Card 3 if NDAMPM>0

Card 3 (I5, E10.3) (Provide one for each of the NDAMPM element groups)

IDMN = element group number.

η_E = material loss factor for current element group.

C8.B Decomposition of Complex Stiffness Matrix

Header Card (A6)

HEADR = 6 character literal constant 'IDCOM8'.

Card 1 (2I5)

IOPT = 1, decomposed stiffness matrix for constrained system.

= 2, decompose $\alpha[M] + [K]$ for free-free or partially constrained system.

IPTC = print control for tabulation of detailed pivot ratio and significance loss calculations. (A summary of pivotal significance loss statistics is always provided.) If IPTC = 0, printout of the table is suppressed; if IPTC > 0, only lines of the table associated with pivotal significance losses equal to or exceeding the integer supplied are printed; if IPTC <0, the entire table is printed.

Card 2 (E10.3) (omit if IOPT ≠ 2)

ALPHA = constant used to overcome problem of ill-conditioning of stiffness matrix for free-free or partially constrained system. The default value is 10.0.

C9.F Natural Frequency Analysis via Direct Iteration Method (IEIGEN = 7)

Header Card (A6)

HEADR = 6 character literal constant 'IEIGN7'.

Card 1 (4I5)

IOPT = 0, complex natural frequencies are computed.

IPTC = 0, printout of normalized eigenvectors (mode shapes) is suppressed.

= 1, normalized eigenvectors are printed out with three significant figures.

= 2, normalized eigenvectors are printed out with eight significant figures.

Card 2 (3I5, E10.3,I5)

NM1 = first mode to be computed. At the first time an analysis is performed, NM1 must be equal to 1. For subsequent runs requesting additional modes, NM1 must be one more than the last mode computed during the previous run.

NM2 = last mode to be computed during this run.

MNIT = maximum number of iterations allowed. The default value is 20.

TOL = tolerance to which iterations are carried out. The default value is 0.001.

UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM
(highest classification of Title, Abstract, Keywords)

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.)

MARTEC Ltd.
Suite 400, 1888 Brunswick St.,
Halifax, N.S. B3J 3J8

2. SECURITY CLASSIFICATION (overall security classification of the document including special warning terms if applicable).

Unclassified

3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title).

Further Enhancement of Frequency Response and Damping Analysis Capabilities of the VAST Finite Element Code (Phase III) Part II: Development of an Out-of-Core Complex Eigenproblem Solver

4. AUTHORS (Last name, first name, middle initial. If military, show rank, e.g. Doe, Maj. John E.)

Jiang, L., Chernuka, M.W.

5. DATE OF PUBLICATION (month and year of publication of document)	6a. NO OF PAGES (total containing information include Annexes, Appendices, etc).	6b. NO. OF REFS (total cited in document)
September 1997	45	10

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered).

DREA Contractor Report

8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address).

Defence Research Establishment Atlantic
P.O. Box 1012
Dartmouth, N.S. B2Y 3Z7

9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant).	9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written).
1.g.c.	W7707-5-3261/01-HAL

10a ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document).	10b OTHER DOCUMENT NOS. (Any other numbers which may be assigned this document either by the originator or by the sponsor).
MARTEC Report No. TR-97-24	DREA CR/98/411

11. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)

(x) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; further distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other (please specify):

12. DOCUMENT ANNOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected).

Full, unlimited

UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM

DCD03 2/06/87-M

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

An effective numerical method for solving complex eigenvalue problems, the complex inverse power method, is discussed in this report. The method is derived by extending its real version to handle complex stiffness matrices resulting from the finite element discretization of the viscoelastic damping materials. Details on the implementation of this method into the VAST program are then presented. Finally, the newly developed complex eigenproblem solver in VAST is verified by numerical problems.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title).

Finite Element
Frequency Response
Vibration
Structural Acoustics
Damping Loss Factor
Constrained Layer Damping
Complex Eigenproblem Solver